2014

MATH 120-01/03 Elementary Functions

David Gerberry

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MATH 120  Elementary Functions - Fall 2014

Instructor:  David Gerberry

Schedule:

120-01:  120-03:
MWF 8-  MWF 11-
8:50am  11:50am
MCD 130  HAI 1


This course requires the use of a TI-84 (or TI-83) graphing calculator. This calculator is standard in all Xavier math courses.

XU Catalog Description:  Graphs and properties of functions, including polynomial functions, exponential functions, logarithmic functions, inverse functions and composition of functions. Applications to real world situations using algebraic, numerical, and graphical methods.

My Description:  We are currently in the midst of what is often referred to as the "information age" where quantitative data on many aspects of life is accumulating like never before. The challenge is being able to make sense of such masses of information. An increasingly useful tool for doing so is modeling. Modeling is the concept of creating a simplified representation of a complex system in order to gain insight into the complex system. Modeling often involves mathematics, more specifically functions. In this course, we study different kinds of functions and will do problems such as "solve for $x$," but our focus will be on how these mathematical ideas can be applied in a variety of areas.

Course objectives:

1. Students will be able to analyze and transform between different representations of a function including graphs, equations and tables.
2. Students will be able to use mathematical language to describe and answer important real-world questions.
3. Students will increase their confidence and competence in mathematical thinking and
GOAL 1: Students will be effective communicators in writing and orally.
2. Students will formulate clear and arguable theses, supported by evidence drawn from appropriate sources.

Goal 1.2 will be achieved through a focus on using quantitative data and mathematical techniques as a means to address real-world problems. Mathematics is a language that is universal and unambiguous, so quantitative data and mathematical writing provides a unique vehicle for effective communication.

GOAL 2: Students will be critical thinkers.
1. Students will analyze and interpret texts, images, objects, artifacts, and quantitative and qualitative data.
5. Students will evaluate the use of science and mathematics in society and everyday life in an informed manner.

Goals 2.1 and 2.5 are central themes of this course.

Grades: We will use the following scale for letter grades:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Score Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>90-100</td>
</tr>
<tr>
<td>B</td>
<td>80-89</td>
</tr>
<tr>
<td>C</td>
<td>70-79</td>
</tr>
<tr>
<td>D</td>
<td>60-69</td>
</tr>
<tr>
<td>F</td>
<td>&lt; 60</td>
</tr>
</tbody>
</table>

Grades will be calculated as follows:

<table>
<thead>
<tr>
<th>Component</th>
<th>Points</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework</td>
<td>150 pts</td>
<td>(20%)</td>
</tr>
<tr>
<td>Quizzes</td>
<td>50 pts</td>
<td>(7%)</td>
</tr>
<tr>
<td>Attendance</td>
<td>50 pts</td>
<td>(7%)</td>
</tr>
<tr>
<td>Exam 1</td>
<td>100 pts</td>
<td>(13%)</td>
</tr>
<tr>
<td>Exam 2</td>
<td>100 pts</td>
<td>(13%)</td>
</tr>
<tr>
<td>Exam 3</td>
<td>100 pts</td>
<td>(13%)</td>
</tr>
<tr>
<td>Final</td>
<td>200 pts</td>
<td>(27%)</td>
</tr>
<tr>
<td>TOTAL</td>
<td>750 pts</td>
<td></td>
</tr>
</tbody>
</table>

Canvas: Canvas will primarily be used to post grades for this course throughout the semester. In addition to individual scores on assignments/quizzes/exams, the grade book in Canvas will display the overall course score throughout the semester as determined by the above grading scale. Class announcements may also be made via Canvas (if this proves to work better than email).

Borderline grades: The main mechanisms for resolving borderline grades are Final Exam scores and Homework/Quiz scores.

1. Final Exam example: If you are between an A and a B and you got an A on the Final Exam, you will get the A for the
course.
2. **HW/Quiz example:** If you are between a C and a B and have a strong HW/Quiz score of 93% (indicating a strong effort in the course), you will get the B.
3. Class attendance and participation can also be used to resolve borderline grades, although in a more subjective way.

It is possible for Student 1 to have a semester grade of 89.84 (with HW/Quiz score of 0 and Final Exam score of 88) and get a B while Student 2 with a semester grade of 89.79 (with HW/Quiz score of 98 and Final Exam score of 93) gets an A.

+/- grades will be used at the instructor's discretion to resolve borderline situations. +/− grades can only benefit the student (i.e. getting a 90% or above guarantees the student an A).

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### Homework and Quizzes:

#### Homework:

Homework will be assigned using WebAssign online homework system. [Information regarding WebAssign](http://www.cs.xu.edu/~david.gerberry/MATH120/) Click [here](http://www.cs.xu.edu/~david.gerberry/MATH120/) to access WebAssign.

**Importance of homework:**

The importance of completing the homework assignments in this course cannot be overstated. Being able to do mathematics (like playing a sport or a musical instrument) requires practice. Exam questions will be very similar to homework questions in both design and difficulty. Therefore, your completed HW assignments are the best study guide for the exams.

**Homework Suggestions:**

1. Make sure that you print off the HW assignments and do your work on the printout itself. Keep your work so that you have something to go back and study when preparing for exams. At the very least, work on notebook paper but keep your work organized. In previous semesters, too many students simply used scratch paper to complete online HW assignments.

2. For each HW assignment that you complete, make a small list of the problems which were the most difficult and maybe a note on why they were difficult. This will be invaluable when studying for exams.

#### Homework Schedule:

1. The online homework assignments for each week will be due on **XXXXX** nights at 11:59pm.

2. At the beginning of each lesson, we will devote class time to answering HW questions that the class has asked to see via the HW survey below. On **XXXXX** and **XXXXX**, we will spend no more that 5 minutes on HW questions (and only if questions have been asked in the survey). On **XXXXX**, we will spend 10-15 minutes on HW questions.
Late homework:

WebAssign does allow you to request extensions on homework assignments. I have enabled automatic extensions if the request is made before XXXXX at 11:59pm. The extensions are for XXXXX days (i.e. gives you until XXXXX to finish the assignment). The penalty for using such an extension is you lose 25% of the credit for all of the problems you complete after the extension. In special limited circumstances, I will consider granting other homework extensions. You must request these manually through the WebAssign system. If you have a valid reason for needing a HW extension (e.g. are sick, family emergency), request the extension manually and let me know this so I can grant an extension with no penalty.

Projects:

There will be at least two small projects assigned this semester. The scores for these projects will be incorporated into your Quiz score.

Quizzes:

Quizzes will be short (usually 10-12 minutes, 1 or 2 questions). We will aim to have approximately 6 or 7 quizzes during the semester.

Make-up quizzes:

Make-up quizzes will not be given. However, your lowest 1 quiz score will be dropped in calculating your final quiz grade.

Math Tutoring Lab: In addition to my office hours, you should take advantage of the Mathematics Tutoring Lab in CLC 419. You can get free (well, sort of free in that you've already paid for it) HW and studying help, individually or in groups, from upper level mathematics students. It is a great place to meet with classmates to work on HW assignments and have someone on hand for any questions that arise. The MTC is open M-R 10am-8pm, F 10am-2pm, Sun 2-8pm and closed on Saturdays. Its phone number is 745-3069.

Collaboration: You are strongly encouraged to work with fellow students on homework assignments. Doing so is an important step in meeting the course objective of "being able to effectively communicate mathematical ideas and difficulties." Keep in mind though, that collaboration on exams is frowned upon. In fact, we call it cheating, so it is very important that collaboration on HW is done in a way that develops your own ability to solve the assigned exercises.

Disability Services: For students with documented disabilities, the Learning Assistance Center provides accommodations such as extended time on exams, reduced distraction testing environment, note-taking assistance, and assistive technology. These services are provided in a positive and encouraging environment which promotes appreciation for diversity and Cura Personalis. If you feel that you may require these services, please contact me or the Learning Assistance Center directly as soon as possible to arrange for appropriate services.

Disclaimer: Things in life happen that are much more important than this course. While the course policies are "set in stone," please inform me if serious issues come up (e.g. family emergencies, health issues, etc.). Under such circumstances, all policies are flexible and we will find a workable solution that lets you deal with what is important and still get a grade that reflects your understanding of the course material. If something does come up, please alert me via email and/or phone as soon as possible so I don't worry or simply assume you are slacking off.

Exam Exam 1 will be on Friday, September 26.

http://www.cs.xu.edu/~david.gerberry/MATH120/
Schedule:  
Exam 2 will be on Friday, October 24.
Exam 3 will be on Friday, November 21.

The Final Exam is on:
Wednesday, December 17 from 8:00-9:50am in MCD 130 for 120-01 (i.e. the 8am class) and
Friday, December 19 from 10:00-11:50am in HAI 1 for 120-03 (i.e. the 11am class).

Course Outline:

Introduction

- What is a function? 
  - Definition of a function
  - Does this correspondence represent a function?
  - What is its domain, range, codomain?
  - Is it 1-1, onto?
  - Does it have an inverse?
  - Calculate inverse of a function given by an equation

Chapter 1: Functions (come in different forms)

- §1.1 Functions given by Formulas
  - Functional notation
  - Distinguishing inputs and outputs of a function
  - Use practical terms to describe the meaning of specific functional notation (e.g. N(5))

- §1.2 Functions given by Tables
  - Average rates of change
  - Estimate function values for input that is not in the table
  - Limiting values

- §1.3 Functions given by Graphs
  - Where is the function increasing, decreasing, concave up, concave down?
  - Where is the maximum/minimum value of the function? What is the value? Where are inflection points?

- §1.4 Functions given by Words
  - Three-step procedure for finding a formula in a word problem
  - Proportions
  - Constant of proportionality
  - Are two things proportional to each other?

Chapter 2: Graphical and Tabular Analysis

- This chapter focuses on calculator skills mainly and is very boring. We skip it and discuss the skills that we need when we need them.
  Here are some YouTube videos for reference.

- §2.1 Tables and trends
  - Use graphing calculator to make a table of values for a function
  - Trends and Limiting values
  - Optimization from a table of values

- §2.2: Graphs
• Getting a proper viewing window
• Seeing limiting values on a graph

• §2.6: Optimization
  • Using calculator to find the maximum/minimum of a function

• §2.4: Solving Nonlinear Equations
  • Crossing-graphs Method (i.e. Two-graph method)

Chapter 3: Straight Lines and Linear Functions

• Review of lines (algebra-style)
  • Find equations of lines that satisfy certain criteria
  • Use algebra to solve linear equations by hand

• §3.1: The Geometry of Lines
  • General idea of rate of change, slope, rise, run, etc.
  • Point-slope form, slope-intercept form

• §3.2: Linear Functions
  • Rates of change (i.e. slopes) in more general problems

• §3.3: Modeling Data with Linear Functions
  • Test to see if data is linear
    • Easy version with evenly-spaced data
    • Version for unevenly-spaced data
  • If so, find the formula for the line
  • What does the slope mean in practical terms?
  • What does the vertical intercept mean in practical terms?

• §3.4: Linear Regression
  • Find the regression line, what do the slope and vertical intercept mean in practical terms?
  • What does plotting the data and the regression line together tell us?

• Summary of Linear Functions
  • What is the central characteristic and what does it look like on a graph?
    • Linear functions have a constant rate of change (i.e. constant slope $m$)
    • On a graph, this means

  • What are the visual clues?
    • Data points are on a straight line.
  • How do we test data?
    • If inputs are evenly-spaced, we see if the outputs are evenly-spaced.
If inputs are not evenly-spaced, we calculate the average rate of change between data points (i.e. the slope).

Chapter 4: Exponential Functions (and Logarithmic Functions)

- §4.1: Exponential Growth and Decay
  - General idea of exponential functions, including: base, growth/decay factors, etc.
  - Being able to convert between units (e.g. go from yearly growth factor to monthly growth factor)
- §4.2: Constant Percentage Change
  - How to go from "3% growth per year" to "growth factor of 1.03 per year"
  - How to go from "3% decay per year" to "decay factor of 0.97 per year"
- §4.3: Modeling Exponential Data
  - Testing to see if data is exponential
    - Test if the data is evenly-spaced
    - Test if the data is unevenly-spaced
  - If the data is exponential, find the exponential model
- §4.5: Logarithmic Functions
  - Richter scale and intensity of earthquakes
  - Logarithmic form and exponential form
    - Calculating values of logarithms without a calculator
    - Solving equations involving logarithms without a calculator
  - Common Logarithm (i.e. log) and Natural Logarithm (i.e. ln)
- Solving Exponential Equations, Law of Logarithms & Logarithms with other bases
  - Using laws of logarithms to solve exponential equations
  - Logarithms with different bases (i.e. not 10 or $e$)
- Connecting Exponential and Linear Data
  - Use properties of logarithm to get a new test for exponential data
  - Get an exponential regression using the natural logarithm
  - Trout example

Figure 3: Semi-logarithmic Plot of Median Survival Time of Brook Trout as a Function of Temperature
Rocket example

§4.4: Modeling Nearly Exponential Data
- Exponential regression
- Be able to explain the practical meaning of the exponential model
  - e.g. "Cost is increasing by 11.6% per year"
- Be able to discern whether an exponential or linear model is more appropriate

Summary of Exponential Functions
- What is the central characteristic and what does it look like on a graph?
  - Exponential functions grow by a constant factor (i.e. multiples)
  - Exponential functions are always concave up (i.e. U-shaped)
Exponential functions are always concave up (i.e. U-shaped) and have a standard formula of $N = Pa^t$.
- Displays exponential growth if $a > 1$ and exponential decay if $a < 1$.
- On a graph, this means

What are the visual clues?
- Looks can be deceiving with exponential functions. There are other types of functions that can look very similar to exponential functions.
- We can take the natural logarithm (i.e. ln) of the outputs and see if that data is linear.

How do we test data?
- If inputs are evenly-spaced, we see if the ratios of the outputs are the same.
- If inputs are not evenly-spaced, we calculate the ratios of outputs and see if they all reflect the same value of $a$.
- See if the ln of the data is linear.

Summary of Logarithmic Functions
- What is the central characteristic and what does it look like on a graph?
  - The input must grow by a factor of 10 (i.e. multiplied by 10) for the output to increase by 1.
  - Logarithmic functions are always concave down (i.e. $\cap$-shaped).
  - Logarithmic functions grow very, very slowly.
  - On a graph, this means

Chapter 5: A Survey of Other Common Functions

- §5.1: Logistic Functions
  - Understand the terms: carrying capacity, maximum sustainable yield, optimum yield level.
  - Understand why the optimum yield level occurs at the inflection point which is also half of the carrying capacity.
  - Be able to find these quantities if given a logistic equation.
  - Be able to find the logistic equation that satisfies given conditions.
  - Logistic regression.

- §5.2: Power Functions
§5.2: Power Functions

- Homogeneity property
- Test to see if data is from a power function (based on properties of logarithms)

§5.3: Modeling Data with Power Functions

- Regression with power functions and connections to linear regression
- Find the power function that passes through two given points
- Brain sizes example

Summary of Power Functions

- What is the central characteristic and what does it look like on a graph?
  - Power functions exhibit the homogeneity property
  - If the input is multiplied by \( t \), the output is multiplied by \( t^k \)
  - Have a standard formula of \( f(x) = cx^k \)
  - On a graph, this means

  ![Graph showing power function relationship between brain mass and body mass.](image-url)

- What are the visual clues?
  - Power functions are very difficult to recognize visually.
- Power functions are very difficult to recognize visually.
  - They can look like exponential or logarithmic functions.
  - The best way to know is to use the test below
    - How do we test data?
      - Plot the ln(inputs) and ln(outputs) and see if the data is linear.
- §5.4: Combining and decomposing functions
  - Using different models for different parts of a function and combining them to model overall behavior
  - Function composition
  - Piecewise functions
- §5.5: Polynomials and Rational Functions
  - Quadratic functions: graphs are parabolas and have minimum or maximum
  - Test to determine if data is quadratic if the data is evenly spaced
  - Quadratic regression
    - Ch2-FL: Optimizing with Parabolas
      - Finding maximum/minimum values of parabolas without a calculator
      - Used that the x-coordinate of the vertex is always x = -b/2a
  - Higher degree polynomials: graphs have multiple changes in direction
  - Test to determine if data is cubic if the data is evenly spaced
  - Cubic and quartic regression
  - Solving equations by factoring, the quadratic formula and/or the two-graph method
- Other possibilities??
  - Chaotic behavior
  - Polynomial Interpolation

When it's all said and done

- An important skill is to determine the appropriate model for different data sets and justify your choice.
  - A review of this skill

Quiz
Solutions, Exam
Solutions, Notes, etc.: Homework
Assignments: Homework assignments will be posted on WebAssign. Information regarding WebAssign
MATH 120: Homework Questions

Pick a homework assignment

Continue »

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MATH 120: General Comments

General comments about the course. They can be left anonymous or you can add your name.
This can be used for anything you would like to say about the course.
Examples could include, "I hate Section 1.3. I am totally confused. I think everyone is.", "I am really upset that you changed the Exam date to next Friday. It doesn't work for me at all.", "The people in the back of the room are distracting/annoying/cheating/whatever. Can you get this in check, please?"

Exam Information

Before the exam

1. Do not forget about the Mathematics Tutoring Lab in CLC 419 as you prepare for exams.
2. Ideas for studying:
   ◦ Summaries at the end of each chapter
     ■ They are short (only a paragraph or two for each section) and give a good overview of the main ideas.
   ◦ Look over lecture notes from class
     ■ Best indicator of what I think is important and am likely to ask on exams
Emphasize connections between topics and motivations that are difficult to see in the HW problems

- Go through HW assignments
  - The most complete way to review for any math exam
  - Exam problems will be similar in format and difficulty to HW problems.
  - Look through the Skill-Building exercises first and practice any problems that do not seem obvious.
  - When you feel confident that you understand the ideas and are able to do the skill-building exercises, move on to the regular Exercises (i.e. longer word problems).

- Review Exercises at the end of each chapter
  - The Review Exercises serve as a good "self-test," since they are problems you have not done before and because there is a mix of different types of problems from different parts of the chapter.
  - I try to include at least one or two problems directly from the Review Exercises on each exam.

During the exam

1. My exams tend to be longish, so be careful that you are managing your time wisely during the exam. If you know how to do a certain problem but simply do not have enough time, explain to me how you would finish (convince me that you know what you are doing and you will get partial credit).

2. On the exams, it is vital that you explain your reasoning. This helps you to get partial credit on problems that you understand but make small mistakes on. Of course, time may be an issue so do not write elaborate explanations until you are sure that you have enough time for them.

3. Remember that a formula alone does not make a mathematical model. You must define what the variables mean in the formula and what units are used.
   - e.g. Is \( t \) the actual year or the years since a certain time (1985 is \( t=5 \))?
   - You should also be aware of the assumptions that go into each type of model.

Final Exam: The Final Exam is cumulative so problems from any of the above topics are fair game. Being a bit more practical though, time constraints limit the variety of problems that I will include. In general, it is safe to focus most of your studying efforts on the topics that we spent the most time on.

A few notes:
- Some problems (particularly the harder ones) will be problems you've seen before on Exams and Quizzes.
- Since there is so much material, the Final will go into topics with less depth (i.e. focus on the standard problems).

The Final Exam is on:

- Wednesday, December 17 from 8:00-9:50am in MCD 130 for 120-01 (i.e. the 8am class) and
- Friday, December 19 from 10:00-11:50am in HAI 1 for 120-03 (i.e. the 11am class).