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Abstract

In this paper, we examine the effect of a possibility of post-election information asymmetry between voters and candidates on pre-election policy platform choices by candidates. We show that this possibility may lead candidates to over commit during election campaign, which may contribute to an ex post moral hazard problem.

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1 Introduction

In this paper, we examine the effect of a possibility of post-election information asymmetry between voters and candidates on pre-election policy platform choices by candidates. We show that this possibility leads candidates to over commit during the election campaign, which may contribute to an ex post moral hazard problem.

Previous studies in electoral competition either assume candidates always keep their campaign promises or that they do not. For example, traditional models of electoral competition, such as the standard Hotelling-Downs framework, assume that candidates are committed to their campaign promises (Wittman (1977), Wittman (1983), Calvert (1985), Groseclose (2001)). Any possibilities of ex post information asymmetry problem is assumed to be non existent, hence any possibilities that the winning candidate may renege on his campaign promises is eliminated. On the other hand, some electoral models such as in Alesina (1988) and citizen candidate models such as Osborne and Slivinski (1996) and Besley and Coate (1997) acknowledge the existence of the ex post information asymmetry problem between voters and elected officials. Therefore, studies along this strain of literature assume that the ex post commitment is not possible. Although these two strands of literature differ on the assumption of whether or not the ex post information asymmetry problem exists, what is common between the two is that these studies implicitly assume that both voters and candidates know the ex post prospect of candidate commitment for certain (whether or not candidates would honor his or her pre-election promises once elected).

However, given that the candidates are opportunistic, it is entirely possible that, during the pre-election period, voters as well as candidates may *not* know the ex post prospect of candidate commitment but only know the likelihood of such commitment.

Consider a candidate who promises a certain fiscal goal by restricting the budget deficit to 2% of the GDP. Implicitly, this is a promise regarding the efficiency of government operations and scope of governmental policies. Suppose further that the personal preference of the candidate is to actually increase state expenditures, therefore his platform commits him to a more conservative fiscal policy than he would actually like. He would, of course, commit voluntarily to win the election. If the candidate is elected and times are such that voters can observe whether or not he kept his campaign promises, then the candidate keeps his promise. However, there is also the possibility of an exogenous event beyond the control of the politician such as a war. In this case, the war would bring certain unavoidable expenses, so it is clear to all that the budget deficit will be bigger. The politician still could, in principle, implement the same level of 'other' state activity that he promised before the election. However, citizens often cannot observe which of the additional expenditures was really war related and which was not. The problem with the observability of war spending makes it unclear to the voters whether or not the candidate implemented his campaign promise. This allows the candidate to essentially implement his most preferred policy.

In summary, what differentiates this paper from previous studies in electoral competition is that in previous studies it is assumed that the post-election state of the world is known to both voters and candidates (so the voters know whether or not the elected candidate would honor his or her campaign promises), however, in this paper, we consider the possibility that the post-election state of the world is unknown (so voters do *not* know whether or not the elected candidate would honor his or her campaign promises). Voters and candidates only know the likelihood of the post-election state of the world.

In order to address the unknown ex post prospect of candidate commitment in our study, we say that the ex post state of the world is *normal* if the state of the world is *simple*. In such state, we say that the information asymmetry problem does *not* exist. Therefore, the voters can observe whether or not the elected politician honors his campaign promises that he announced during his election campaign. In the case of reneging, the leader must pay a high political price and this entirely discourages such behavior. On the other hand, the ex post state of the world is said to be *non-normal* if the state of the world is *complex*. In such state, we say that the information asymmetry problem exists. In such state, cost of observation for the voters may be very high, and therefore, such observation may not be possible. Hence, reneging is entirely possible in *non-normal* state.

Under this setup, we show that when the ex post state of the world is likely to be *non-normal*, candidates are *more* likely to make a bigger campaign promise on an issue than when the ex post state of the world is likely to be *normal*. Therefore, we show that a possibility of existence of ex post information asymmetry problem leads to candidates making bigger campaign promises, hence the presence of potential ex post information asymmetry may give rise to an ex post moral hazard problem. We also present previous empirical findings that support this hypothesis.

The paper proceeds as follows. In section 2, we present the model of the two candidate electoral competition. In section 3, we present the results of the model and provide a discussion of the results. Section 4 concludes the paper.

2 The model

Consider a two-candidate, single-issue electoral competition where the candidates are policy-motivated. Two candidates, Candidate A and B, are assumed to be a left-winger and a right-winger, respectively. The median voter knows the policy preferences of both candidates which are exogenously given. A policy is a number $z \in \mathbf{R}$. During an election, the candidates announce their policy platforms on the issue. The outcome of the election is determined by the median voter who votes sincerely to maximize her post election utility. The candidate who wins the election implements a policy.

We index the voters $j = 1, 2, ...n$. The voter's utility over the policies of any voter j is $v_j(z)$. We assume that (i) $v_j(z)$ follows a utility function with a unique maximum at some value $\hat{z}_j$ (we will assume that $v_j(\hat{z}) = 0$) and is symmetric around the maximum, and (ii) $v_j(z)$ is differentiable and strictly concave. Furthermore, we assume that the median of the values of $\hat{z}_j$ among the electorate is 0.

The candidates also have utility functions over the policy. Candidate i's utility function is $u_i(z)$ where $i \in \{A, B\}$. We assume that (i) $u_i(z)$ follows a utility function with a unique maximum at some value y_i (we will also assume that $u_i(y_i) = 0$) and is symmetric around the maximum, and (ii) $u_i(z)$ is differentiable and strictly concave. Furthermore, given the two candidates A and B, we assume y_A and y_B (in which they are determined exogenously) are located on the opposite side of the median voter's ideal point. That is, $y_A < 0 < y_B$.

The strategic choice in this electoral competition game are Candidate A's choice of a platform and Candidate B's choice of a platform. These platforms will be denoted as x_A and x_B. Candidate $i \in \{A, B\}$'s platform is interpreted to be a promise to implement policy x_i once elected. We denote $q \in (0, 1)$ as the probability that the ex post state (a state after the winning candidate is chosen) is normal. As discussed earlier, we say that the ex post state of the world is said to be *normal* ($q = 1$) if the voters can observe whether or not the elected politician honored the campaign promises he announced during his election campaign. On the other hand, the ex post state of the world is said to be *non-normal* ($q = 0$) if such observation is not possible. If candidate i wins, he would implement the policy x_i with probability q and he would implement his own ideal policy y_i with probability $(1 - q)$.

Therefore the expected payoff to any voter j given the probability of ex post normal state q is,

$$V_j(x_i, y_i, q) = qv_j(x_i) + (1 - q)v_j(y_i) + \epsilon_i \text{ for all } i \in \{A, B\}. \tag{1}$$

where $\epsilon_i \in \mathbf{R}$ is a candidate specific random shock that is normally distributed with the mean at zero. This randomness basically implies that the election outcome is not only determined by candidates' election strategies but is also influenced by factors other than what the candidates can control.

3 Results

In this section, I characterize the equilibrium platform choices of the electoral competition. Given a choice of platforms (x_A^*, x_B^*) and q and y_i for all $i \in \{A, B\}$, Candidate A wins the election if and only if

$$V(x_A, y_A, q) + \epsilon_A > V(x_B, y_B, q) + \epsilon_B. \tag{2}$$

Equation (2) provides the election rule, and it simply states that a candidate who provides a higher utility to the median voter wins the election. Now, given the election rule and by letting $\epsilon = \epsilon_A - \epsilon_B$, we have

$$\{(x_A, x_B)|\ \epsilon > V(x_B, y_B, q) - V(x_A, y_A, q)\}$$

as a set of equilibrium strategies in which Candidate A wins the election, and we have

$$\{(x_A, x_B)|\ \epsilon \leq V(x_B, y_B, q) - V(x_A, y_A, q)\}$$

as a set of equilibrium strategies in which Candidate B wins the election. Furthermore, we will denote $d = V(x_B, y_B, q) - V(x_A, y_A, q)$ where d simply measures a gap between the median voter's utility gain from the two candidates. Since the random factor ϵ is a continuous random variable, the probability that Candidate B wins the election is

$$Prob(\epsilon \leq d) = \int_{-\infty}^{d} f(\epsilon)d\epsilon = F(d)$$

and the probability that Candidate A wins the election is

$$Prob(\epsilon > d) = 1 - \int_{-\infty}^{d} f(\epsilon)d\epsilon = 1 - F(d)$$

where $F(\cdot)$ is a *cumulative distribution function.*

3.1 Candidates' Expected Utility Maximization

Under the setup discussed above, Candidate i maximizes the following expected payoff by choosing platform x_i during election.

$$\max_{x_i}\ qu(x_i, y_i)prob\,(i \text{ wins}) + (qu(x_j, y_i) + (1 - q)u(y_i, y_j))\,prob\,(j \text{ wins}) \text{ where } i \neq j. \tag{3}$$

The first term in equation (3) measures Candidate i's utility gain given the probability of Candidate i winning the election and the second term measures Candidate i's utility gain given the probability of Candidate j winning the election.

Therefore, Candidate A maximizes the following expected payoff by choosing platform x_A

$$\max_{x_A} \; qu(x_A, y_A)\,(1 - F(d)) + (qu(x_B, y_A) + (1-q)u(y_A, y_B))\,F(d)$$

and Candidate B maximizes the following expected payoff by choosing x_B

$$\max_{x_B} \; qu(x_B, y_B)F(d) + (qu(x_A, y_B) + (1-q)u(y_A, y_B))\,(1 - F(d))\,.$$

At the equilibrium platform choices (x_A^*, x_B^*), the following first order condition must be satisfied for Candidate A:

$$\Phi_A = q\frac{\partial u(x_A, y_A)}{\partial x_A}(1 - F(d)) + \frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_A}[qu(x_B, y_A) + (1-q)u(y_A, y_B) - qu(x_A, y_A)] = 0. \tag{4}$$

Similarly, for Candidate B, at the equilibrium platform choices (x_A^*, x_B^*), the following first order condition must be satisfied:

$$\Phi_B = q\frac{\partial u(x_B, y_B)}{\partial x_B}F(d) - \frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B}[qu(x_A, y_B) + (1-q)u(y_A, y_B) - qu(x_B, y_B)] = 0. \tag{5}$$

3.2 Symmetric equilibrium

In this section, we examine an electoral competition where two candidates have symmetric preferences around the median voter. That is, $|y_A| = |y_B|$. Although this assumption may appear to be too restrictive, it is not altogether unreasonable. For example, in U.S. politics, Republicans and Democrats are generally viewed as having symmetric preferences.

Lemma 1. Implicit Function Theorem *Assume that the candidates have symmetric preferences around the median, then there exists a continuously differentiable function* $x = x(\mathbf{z})$ *on an open ball* B *about* $\mathbf{z}^*$.

Proof. See Appendix. □

Proposition 1. *Assume that the candidates have symmetric preferences around the median, then* $\frac{\partial x_A(\mathbf{z}^*)}{\partial q} < 0$ *and* $\frac{\partial x_B(\mathbf{z}^*)}{\partial q} > 0$.

Proof. See Appendix. □

Proposition 1 implies that, in equilibrium, when the ex post state of the world is likely to be *non-normal*, candidates choose to make bigger policy promise by choosing policy positions closer to the median voter and away from the candidates' own preferred policies. On the other hand, when the ex post state of the world is likely to be *normal*, candidates choose policy positions that are more true to their own policy preferences. The intuition behind this result is straight forward. When there is a greater likelihood that the ex post state is *non-normal*, the chance of being free from keeping campaign promises is high, therefore, the candidates offer policies that are more appealing to the median voter at the cost of moving away from their own policy preferences.

Several findings from previous empirical studies support this result. I present two examples here. The first example deals with issue complexity (simple v.s. complex). Carmines and Stimson (1980) find evidence that candidates typically have unambiguously distinctive policy positions on what is perceived as easy issues by the electorate. However, candidates tend to take similar policy positions on what is perceived as hard issues by the electorates. In their study, Carmines and Stimson defined easy issues as simple and symbolic whereas hard issues are defined as complex and technical. As discussed earlier, when an issue is more complex to understand, the ex post information asymmetry problem is more likely to arise. Within our context, this implies candidates are more likely to make excessive commitments during the election campaign and choose policy platforms that are closer to the median, hence their proposed policy positions would be similar. On the contrary, when an issue is relatively easy to understand, the ex post information asymmetry problem is less likely to arise. Within our context, this implies candidates are less likely to make excessive commitments during election campaign and choose policy platforms that are more closer to their own preferred positions away from the median, hence their proposed policy positions would be unambiguously distinct.

The second example deals with voter trust between local government v.s. federal government. Several studies show that the public holds state legislatures to be more honest and caring than the national congress (Newkirk (1979) and Jewell (1982)). Similar studies find that local governments are also viewed favorable in contrast to the federal government and city government officials are seen as much more honest than those in the national government (Ulbig (2002)). The public feels this way because the public has more control over the actions of elected officials at the local level (Dahl and Tufte (1973) and Diamond (1999)). Within our context, this implies that the rise of information asymmetry problem is minimized at the local level, because the public has greater control over the actions of elected officials. If indeed voters feel distanced with the issues involving national government and has little or no control over actions of elected officials at the national level, the information asymmetry problem is more likely to arise. In other words, because national issues are more likely to be complex than local issues, candidates are more likely to make bigger campaign promises on national issues, making the moral hazard problem more likely to arise which in turn leads to the loss of credibility of politicians.

4 Concluding remarks

In this study, we highlight the importance of ex post information symmetry between voters and elected officials in a public choice sense. If political candidates perceive the future state of the world to be more transparent, this would discourage candidates from making bigger campaign promises, hence minimize the rise of ex post moral hazard problem.

One can easily extend our model to further the study on electoral competition. For example, it is assumed in this paper that voters have full information about the candidates' most preferred policies. Although it is not so unreasonable to assume that voters know the candidates' preferred policy positions, it would be worthwhile to consider a case where voters have incomplete information about the candidates' most preferred policy positions. Furthermore, although some previous empirical findings are provided in this study that supports our hypothesis, one could test the result empirically as a follow-up study.

5 Appendix

Proof of Lemma 1. Let us first define a vector $\mathbf{z} = (y_i, y_j, q)$ for $i \in \{A, B\}$. In order to prove Lemma 1, we need to show that the following partial Jacobian matrix is nonsingular.

$$\begin{pmatrix} \frac{\partial \Phi_A}{\partial x_A} & \frac{\partial \Phi_A}{\partial x_B} \\ \frac{\partial \Phi_B}{\partial x_A} & \frac{\partial \Phi_B}{\partial x_B}. \end{pmatrix} \tag{6}$$

Now from the first order condition in equation (4), we have

$$\frac{\partial \Phi_A(\mathbf{z}^*)}{\partial x_A} = q\frac{\partial^2 u(x_A, y_A)}{\partial x_A^2}(1 - F(d)) - 2q\frac{\partial u(x_A, y_A)}{\partial x_A}\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_A}$$

$$+\frac{\partial\left(\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_A}\right)}{\partial x_A}\left(qu(x_B, y_A) + (1-q)u(y_A, y_B) - qu(x_A, y_A)\right).$$

With a simple algebraic manipulation (by multiplying x_A to the equation above and dividing it by x_A) and applying the *envelope theorem*, we can rewrite the above equation as

$$\frac{\partial \Phi_A(\mathbf{z}^*)}{\partial x_A} = \Phi_A(\mathbf{z}^*) - 2q\frac{\partial u(x_A, y_A)}{\partial x_A}\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_A} < 0. \tag{7}$$

Notice that from the first order conditions, $\Phi_i(x_A^*, x_B^*, \mathbf{z}^*) = 0$, hence it is easy to see that $\frac{\partial \Phi_A(\mathbf{z}^*)}{\partial x_A} < 0$

Similarly, we can show that

$$\frac{\partial \Phi_B(\mathbf{z}^*)}{\partial x_B} = \Phi_B(\mathbf{z}^*) + 2q\frac{\partial u(x_B, y_B)}{\partial x_B}\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B} < 0. \tag{8}$$

Also, from the first order condition in equation (4), we have

$$\frac{\partial \Phi_A(\mathbf{z}^*)}{\partial x_B} = -q\frac{\partial u(x_A, y_A)}{\partial x_A}\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B} + q\frac{\partial u(x_B, y_B)}{\partial x_B}\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_A}$$

$$+\frac{\partial\left(\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_A}\right)}{\partial x_B}\left(qu(x_B, y_A) + (1-q)u(y_A, y_B) - qu(x_A, y_A)\right).$$

Now, from the above equation, notice that $\frac{\partial\left(\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_A}\right)}{\partial x_B} = 0$, hence

$$\frac{\partial \Phi_A(\mathbf{z}^*)}{\partial x_B} = -q\frac{\partial u(x_A, y_A)}{\partial x_A}\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B} + q\frac{\partial u(x_B, y_A)}{\partial x_B}\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_A} > 0 \tag{9}$$

Similarly, we can show that

$$\frac{\partial \Phi_B(\mathbf{z}^*)}{\partial x_A} = q\frac{\partial u(x_B, y_B)}{\partial x_B}\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_A} - q\frac{\partial u(x_A, y_B)}{\partial x_A}\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B} > 0. \tag{10}$$

Given the Jacobian matrix in (6), for it to be singular, it must be that

$$\frac{\partial \Phi_A}{\partial x_A}/\frac{\partial \Phi_A}{\partial x_B} = \frac{\partial \Phi_B}{\partial x_A}/\frac{\partial \Phi_B}{\partial x_B}. \tag{11}$$

Note that under the symmetric assumption, we have that $\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_A} = \frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B}$. Now, lets's suppose that indeed $\frac{\partial \Phi_A}{\partial x_A}/\frac{\partial \Phi_A}{\partial x_B} = \frac{\partial \Phi_B}{\partial x_A}/\frac{\partial \Phi_B}{\partial x_B}$ (hence, the Jacobian matrix is singular). Then, by applying the symmetric assumption, we have

$$2\frac{\partial u(x_A, y_A)}{\partial x_A}/\left(\frac{\partial u(x_A, y_A)}{\partial x_A} - \frac{\partial u(x_A, y_A)}{\partial x_B}\right)$$
$$= \left(\frac{\partial u(x_A, y_A)}{\partial x_A} - \frac{\partial u(x_A, y_A)}{\partial x_B}\right)/2\frac{\partial u(x_A, y_A)}{\partial x_A}.$$

Given the equation above, the only way that the condition given by equation (11) can hold is if

$$2\frac{\partial u(x_A, y_A)}{\partial x_A} = \frac{\partial u(x_A, y_A)}{\partial x_A} - \frac{\partial u(x_A, y_A)}{\partial x_B}$$
$$\Rightarrow \frac{\partial u(x_A, y_A)}{\partial x_A} = -\frac{\partial u(x_A, y_A)}{\partial x_B}.$$

This is a contradiction since $\left|\frac{\partial u(x_A,y_A)}{\partial x_A}\right| < \left|-\frac{\partial u(x_A,y_A)}{\partial x_B}\right|$. Hence, the condition given by (11) cannot hold, and therefore, the Jacobian matrix is nonsingular.

Proof of Proposition 1

Part 1 We first establish that, under our setup, candidate's equilibrium policy platforms, x_A^* and x_B^* diverge around the median. That is, $y_A < x_A^* < 0 < x_B^* < y_B$. In part 2, we will prove the result in proposition 1.

In order to show this divergent result, we first show that (x_A^*, x_B^*) is unique and bracket around the median voter's most preferred policy position.

To show that (x_A^*, x_B^*) is unique, consider Candidate i. Given x_j^*, assume that at x_i^*, $EU_i = EU_i^*$ where $i \neq j$. Now suppose there exist $|x_i'| \geq |x_i^*|$ such that at x_i', $EU_i = EU_i^*$. Then it must be the case that $x_i' = x_i^*$. If not, Candidate i is always better off choosing x_i^* because $v(x_i^*) > v(x_i')$.

Now we show that candidates' equilibrium platforms bracket around the median voter's most preferred position. We show this in four steps.

First, we show that $x_i^* \neq 0$ for all $i \in \{A, B\}$. Now, suppose $x_i^* = 0$. Recall that $d = qv(x_i) + (1-q)v(y_i) - qv(x_j) - (1-q)v(y_j)$ for $i \neq j$, and therefore, $\frac{\partial d}{\partial x_i} = q\frac{\partial v(x_i)}{\partial x_i}$. Notice that $\frac{\partial v(x_i)}{\partial x_i} = 0$ at $x_i^* = 0$ ($x_i = 0$ is the most preferred position for the median). Hence, at $x_i^* = 0$, the first order condition given by equation (4)[1] becomes

$$q\frac{\partial u(x_i^*, y_i)}{\partial x_i}(1 - F(d)) = 0$$

which is a contradiction since $\frac{\partial u(x_i^*,y_i)}{\partial x_i} < 0$. Hence $x_i^* \neq 0$, for all $i \in \{A, B\}$.

Second, we show that $x_i^* \in \mathbf{R}_-$ for $i = A$ and $x_i^* \in \mathbf{R}_+$ for $i = B$. Let's suppose that $x_i^* \in \mathbf{R}_-$. Also, suppose that there exists $x_i' \in \mathbf{R}_+$ such that $|x_i^*| = |x_i'|$ and without loss of generality, we assume that $\epsilon = 0$. Then, it must be the case that

$$prob(\text{i winning at } x_i^*) = prob(\text{i winning at } x_i').$$

[1]This is a case where $i = A$. We get the similar result for using the first order condition given by equation (5). In that case $i = B$.

However, since $y_i \in \mathbf{R}_+$ for $i = B$, $\frac{\partial u(x_i,y_i)}{\partial x_i} > 0$, $\frac{\partial^2 u(x_i,y_i)}{\partial x_i^2} < 0$ and $\frac{\partial u(x_i,y_i)}{\partial x_i}|_{(x_i=y_i)} = 0$,

$$u(x_i^*) < u(x_i').$$

This is a contradiction, hence $x_i^* \notin \mathbf{R}_-$ for $i = B$. The other case where $x_i^* \in \mathbf{R}_+$ for $i = B$ can be shown similarly, hence is omitted.

Third, we show that $x_i^* \neq y_i$. Suppose that $x_i^* = y_i$ then, given the properties of $u(\cdot)$, $\frac{\partial u(x_i,y_i)}{\partial x_i} = 0$. Therefore, the first order condition given in equation (5) can be rewritten as

$$-\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B}\left((1-q)u(y_A,y_B) - qu(x_B,y_B)\right) = 0.$$

However, since $\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B} > 0$, it must be that

$$-\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B}\left((1-q)u(y_A,y_B) - qu(x_B,y_B)\right) < 0.$$

Hence, it cannot be the case that $x_i^* = y_i$. The case where $i = A$ is similar, hence is omitted.

Lastly, we show that, in equilibrium, $|x_i^*| < |y_i|$ for all $i \in \{A, B\}$. Again, we show the case where $i = B$ and omit the case where $i = A$. Let $|x_B^* - y_B| = |x_B i' - y_B|$ and also let's suppose that $x_B^* < y_B$ and $x_B' > y_B$, then $u(x_B^*, y_B) = u(x_B', y_B)$. However,

$$Prob(i \text{ winning at } x_B^*) > Prob(i \text{ winning at } x_i')$$

Therefore, Candidate B is always better off choosing x_B^*. Hence, $|x_B^*| < |y_B|$.

Part 2 Now for Proposition 1 to hold, it must be that

$$\frac{\partial x_A(\mathbf{z}^*)}{\partial q} = -\frac{\frac{\partial \Phi_A(\mathbf{z}^*)}{\partial q}}{\frac{\partial \Phi_A(\mathbf{z}^*)}{\partial x_A}} < 0 \text{ and } \frac{\partial x_B(\mathbf{z}^*)}{\partial q} = -\frac{\frac{\partial \Phi_B(\mathbf{z}^*)}{\partial q}}{\frac{\partial \Phi_B(\mathbf{z}^*)}{\partial x_B}} > 0.$$

Recall that lemma 1 insures the existence of a continuously differentiable function $x = x(\mathbf{z})$ on an open ball B about $\mathbf{z}^*$ where $\mathbf{z} = (y_i, y_j, q)$.

We prove that $\frac{\partial x_B(\mathbf{z}^*)}{\partial q} > 0$ here. Proof for $\frac{\partial x_A(\mathbf{z}^*)}{\partial q} < 0$ is similar, hence is ommited. Now first notice that $\frac{\partial \Phi_B(\mathbf{z}^*)}{\partial x_B} < 0$, therefore we need to only show that $\frac{\partial \Phi_B(\mathbf{z}^*)}{\partial q} > 0$ to show $\frac{\partial x_B(\mathbf{z}^*)}{\partial q} < 0$. From the first order condition given in equation (5), we have

$$\begin{aligned}\frac{\partial \Phi_B(\mathbf{z}^*)}{\partial q} &= \frac{\partial u(x_B,y_B)}{\partial x_B}F(d) + q\frac{\partial u(x_B,y_B)}{\partial x_B}F(d)\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial q} \\ &-\frac{\partial\left(\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B}\right)}{\partial q}\left(qu(x_A,y_B) + (1-q)u(y_A,y_B) - qu(x_B,y_B)\right) \\ &-\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B}\left(u(x_A,y_B) - u(y_A,y_B) - u(x_B,y_B)\right).\end{aligned}$$

By multiplying q to the equation above and using the *envelope theorem*, we have

$$\begin{aligned}\frac{\partial \Phi_B(\mathbf{z}^*)}{\partial q} &= \Phi_B(\mathbf{z}^*) + q^2\frac{\partial u(x_B,y_B)}{\partial x_B}F(d)\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial q} \\ &-q\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B}\left(u(x_A,y_B) - u(y_A,y_B) - u(x_B,y_B)\right).\end{aligned}$$

where we know $\Phi_B(\mathbf{z}^*) = 0$. Now, given the symmetric assumption,

$$\frac{\partial d}{\partial q} = v(x_B) - v(y_B) - v(x_A) + v(y_A) = 0,$$

hence

$$\frac{\partial \Phi_B(\mathbf{z}^*)}{\partial q} = -q\frac{\partial F(d)}{\partial d}\frac{\partial d}{\partial x_B}\left(u(x_A, y_B) - u(y_A, y_B) - u(x_B, y_B)\right) > 0.$$

Therefore $\frac{\partial x_B(\mathbf{z}^*)}{\partial q} > 0$.

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