MATH 370-01 Introduction to Real Analysis

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Math 370  Introduction to Real Analysis, Spring 2014
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1. OFFICE HOURS
My office is in Hinkle Hall, ground floor, Room 132. My office hours are TR 2:00-3:30, and more hours may be added later. The hour before and after our clas meetings will also be good times for us to talk. Feel free to stop by my office at any time during the semester. If the door is closed, please knock! If I am there, I will be happy to meet with you! I check e-mail frequently during the day; e-mail is the best way to set a meeting time. For conversations about mathematics I strongly prefer to meet with you in person.

2. CONTENT
We will work through a sequence of definitions and theorems, leading from basic properties of real numbers to the Fundamental Theorem of Calculus in such a fashion that the students in the class will understand what the theorems are saying and why they are true.

3. GOALS
Successful students will practice and learn to think logically, as mathematicians do, and they will learn and practice to present and to write up their ideas and results. You, being one of them, will learn to share your ideas so that an audience can follow you, to defend your ideas when necessary, and also to recognize and admit that you have made a mistake, if that is the case. These skills are useful in industry, and in every walk of life. For many of you these skills may be more important than the content of the course, introductory Real Analysis. In order to do and to communicate mathematics it is crucial to learn to use language very carefully, when you present mathematics to others, when you listen to others present or when someone asks you a question, when you write mathematics, and when you ask a question.

I hope to convince you that doing mathematics, proving that theorems are true, is fun and exciting. And I believe that it is not possible to help you learn this central part of mathematics by lecturing. Just as you cannot learn to ride a bicycle or learn to swim by listening to a lecture, I believe that you can not learn how to prove theorems by watching someone do it. I believe that the only way for you to learn to prove theorems is for you to prove theorems yourself. I am also convinced that you will learn the material much more deeply and more lastingy if you, as individuals and as a group, create and scrutinize the arguments yourself rather than listening to lectures about the results of what others have done. Instead of following a guide through a gallery of magnificent paintings, you will learn how to paint yourself, a distinctly different experience, which is much more satisfying and valuable!

4. HOW THE COURSE WILL BE CONDUCTED
I will try to find theorems that are easy enough so that you can prove them without the aid of "sample proofs", but hard enough that you can be proud about finding a proof, and which will lead us through the described content area. As we progress, the problems may get a bit harder, but if you understand the earlier ones the later ones will not be more difficult to prove. Your skills will improve. You will, I hope, find that working these problems is like working puzzles - challenging and fun and very satisfying when you solve them. You will need to think about how to prove a Theorem for an extended period, not just once and expect to get it done. Finding a proof takes time, and cannot be forced. I do not expect each of you to prove every theorem/solve every problem. I do expect that each of you TRIES to solve each problem yourself, and that you make sure that you completely understand each proof after it has been presented.

The heart of the course is students presenting their arguments/proofs in class, with the goal to convince the (critical) audience that you have found a valid argument which proves that the Claim of the Theorem
you wanted to prove is true. At the same time, when someone is presenting, your job is to make sure that you can follow (and later reproduce) the argument. That means that your job is to ask all questions you need answered to clarify the argument presented until you can follow it completely, so that you could go through it yourself. In this way the class community will make sure that the presented argument is logical and valid. Before we move on, everyone must be convinced that we have a proof, without any doubt left. **However:** You are NOT to suggest alternative routes or arguments that might also prove the theorem - your job is to follow the presenter's argument, and if it is not clear to you how or why something follows, you must ask the presenter for clarification. NOTE: In my office, I will be happy to go through any of the arguments that were presented in class, at any time, and as often as you want.

You should attempt every problem. Even if you do not solve a problem but have worked on it, you are much more likely to understand a proof presented in class by one of your peers. After each meeting, you must write up a proof for each of the problems presented, so that you have a complete record at the end of the course. We will take pictures of the blackboard at the end of each presentation, and I will post these pictures on our class web-site: cerebro.cs.xu.edu/math/math370/01s13/calendar.html.

5. GRADING

**Midterm and Final exams** together will make up 20% of your course grade.
**Weekly write-ups** will account for 30% of your course grade. (more details below.)
**Presentation/Participation Grade** will count for 50% of your course grade. (more details below.)
**Portfolio** Your course grade will be lowered by up to 1.5 letter grades if you do not maintain a portfolio.
**Absences** If you miss classes excessively, I will lower your course grade.

**Midterm and Final Exams**
We will take two exams: An in-class mid-term exam, some time after the course is half over, and a final exam held at (or due on) the officially scheduled time.

**Weekly write-ups**
Every Monday, you will hand in a written proof for one of the problems from the problem sequence, giving a detailed, logically sound and correct argument using complete, unambiguous, and proper English sentences and/or standard mathematical symbols. Your weekly submission is to be labeled as "TURN IN" at the top of the page along with your name, the date, the problem number, and the problem statement. The solution/proof should be neatly written and double spaced to allow room for my comments.

It is YOUR decision which problem you write up and submit each week. In the beginning, you should pick a problem that has been presented in class, which you think you understand well. Later in the course, you may decide (or I may ask you) to write up problems that have not yet been presented in class. The problem write-up you hand in will be graded, and I will give feedback on the writing as well as the logic and correctness of your proof. This important component of the course is designed to help you improve your writing style and the clarity of your written arguments. It is important that you choose to write up problems that challenge your writing abilities, but you must be sure that you understand the argument. Grades for write-ups will be based on the following rubric:

- **A:** Your solution is correct and well-written. It is not difficult to read and follow your argument.
- **B:** I am convinced that you know how to do the problem but something is missing in the write-up, or something you wrote is not correct, or is ambiguous, or does not follow as you say it does.
- **C:** You may know how to do the problem but I cannot be sure of that from what you are writing.
- **D:** There is at least one major flaw in your argument.
- **F:** I cannot find any useful ideas in what you wrote towards proving the Claim - or - you did not submit a problem this week.

If you get a grade below B+ (but above F) on a Turn-In, then you can re-write the same problem and turn
it in on the following Monday, clearly marked as "RE-WRITE". I will grade it again, and only count the better of the two grades, but B+ is the best you can get on a RE-WRITE. There should be at most one re-write for each problem - but we may communicate about making an exception. However, you must turn in one new problem each week. I will not accept late work. If you have to miss class you can have someone bring your work to class, or e-mail your write-up to me before class. I will grade your problem only if it has been submitted by the time our class meets, but not if it is submitted late.

As a general rule, you may discuss the problems with other students, in fact, I encourage you all to discuss mathematics with each other!!! But once you decide to write up and turn in a certain problem, I expect that you will not discuss that particular problem further with anyone but me, until it has been graded and returned. If you want help deciding which problem to write up, I encourage you to come and talk to me. After you have started to work on a problem, you may come to talk to me about it in my office.

**Your Presentation and Audience Participation**

This is the most important component of the course, and therefore carries the most weight in terms of grades. For each problem, the name of one of you is called. If your name is called, you have the opportunity to present your proof for the problem at hand. You also have the option to "pass" and the next name will be called. If you begin your presentation and a flaw is discovered, or if you cannot answer a question from the audience in a satisfactory way, your presentation will stop. You may then pass the problem to someone else, or you may keep it until the next meeting. If you keep the problem, you will try to present a corrected proof/solution at the beginning of the next meeting. Presentations rubric:

- 3 points: The audience understands your argument and accepts it as correct
- 2 points: Your argument did not pass last time, but your revised argument does.
- 1 point: The audience does not accept your argument as correct
- 0 points: You have no useful idea, but you claimed to have a solution/proof

Your participation as member of the audience is very important, and I will do my best to keep track of the frequency and quality of your contributions as audience. Your job is to carefully follow and review every step the presenter makes, and to make sure that the next conclusion follows from whatever had been established before. The question is usually not "Is the statement that was concluded true?" The question is usually: "Does its truth follow?" All of the Theorems we prove are true statements. The goal is to show how their truth can be established!!! (That is what a proof does.) While it is more difficult for me to keep track of your audience participation, it will figure into this portion of the overall grade, especially if it is particularly strong, or if it is particularly poor.

**Portfolio**

You must maintain an up-to-date portfolio of all proofs which have been presented in class. The portfolio should contain the Problem numbers, problem statements, and (reasonably) complete proofs. You need to make sure that you understand the arguments in class well enough to write at least a good outline of the argument afterwards. I may check if your portfolio is up to date at any time during the semester. Not having a complete and useful portfolio at the end of the semester results in a serious penalty: Your semester grade will be lowered by at least one letter grade, up to 1.5 letter grades, if the portfolio is not complete.

**6. GENERAL COMMENTS**

I will almost never lecture, except perhaps in response to a question, to introduce a new concept or definition, or if I feel some clarification is useful or needed. I urge you to talk with me often, about the problems you are working on. It is my goal and desire to help you as much as I can! But you have to allow me to try to help you in a way that I consider to be most useful, even though you may disagree with my methods at the time. You may think that the best way for me to help you is for me to tell you how to do something. (If this is so, then I will disagree.) But I may just talk with you briefly, ask a few questions,
trying to figure out what you are thinking, and then ask you to go and think about what we said. You will feel the greatest satisfaction if YOU solved the problem yourself! And to get you there is always my goal.

The method of instruction I use in this course is called "Moore Method", named after R.L. Moore. Moore was a world-famous topologist, who taught all his courses without a text and he almost never lectured. He simply assigned problems and his students presented their solutions at the board in class. He forbid his students to discuss problems with each other or to read in any books, until they were doing research for their PhD theses. I do not restrict (y)our work in this way.

Before you go to the board to present YOUR solution, you need to anticipate questions from the audience, and you need to be prepared to go into more detail when asked by the audience. Question your own proofs before presenting. Try to learn to analyze arguments objectively, and to leave your ego out of it. Don't be too quick to think that you are wrong if someone asks a question, but be open to recognizing that you made a mistake when it happens. Simply try to learn from it.

7. IMPORTANT RULES

The person at the board is presenting his/her argument to prove a certain statement based on certain assumptions. It will be important that the presenter starts with BOTH, the statement to be proved and the statements which are assumed to be true (assumptions). The presentation is a logically correct argument which leads from the assumptions to the claim. If someone in the audience cannot follow a certain step, it is that person's job to ask the presenter for clarification. This is to be done politely. If the presenter cannot justify the conclusion, then the presentation is over. The presenter will choose to keep the problem for next time, or allow someone else to present the problem.

The presenter should leave time and stop for the audience to ask questions after each step in the argument.

Do not suggest alternative ways to solve a problem, unless the class decides to do that after a presentation. The audience should only ask for clarification of something that has been presented.

**Words we don't use, and things we don't do.**
- We don't let arguments go by which we don't understand.
- We don't try to put up solutions/proofs we do not feel confident about.
- We don't present vague arguments. Wrong arguments can be corrected. Vague arguments cannot.
- We don't use the word "obvious".
- We don't use, or think, the word "stupid".
- We don't attack people personally.
- We don't try to intimidate people who ask questions of our arguments.
- We don't get mad. If we do, we get over it fast.
- We don't get upset with ourselves or with each other, when a mistake has been made.
- We don't use concepts we have not defined

**Words we use, and things we do.**
- We understand our own arguments fully, so that it will be as clear as possible at the board.
- We present our arguments in a step by step form to make it possible for others to analyze them.
- We ask questions of the form: "Could you explain why "this" follows from "that" ...
- We give people time to answer the questions we have asked.
- When we use an earlier result in our argument, we cite it specifically, e.g.: "By Problem 29, it follows that..."
- We try to learn from our mistakes.
- We let people know when we think they have done something clever.